BC Calculus Summer Work 2018

Below you will find an assignment set based on the prerequisites needed for the BC Calculus curriculum and your future mathematical studies. The problems assigned should be the minimum you do to prepare for this course next year. It is expected that you:

1. Answer all questions on a separate sheet of paper with all work shown. Each problem must be completed and your work is due on the first day of class.

2. Be ready for a calculator/no-calculator assessment on all materials below within the first term.

3. Review Chapters 1-7 of the AP Calculus Interactive Lectures by Rita Korsunsky (ibook). These chapters cover the topics to be completed during the first term of class.

NO CALCULATOR PORTION

1. Sketch by hand a variety of transformations involving exponential, logarithmic, and/or trigonometric functions.
   A. \( y = 3^{-x} + 2 \)  
   B. \( y = \log_4(x + 2) - 1 \)  
   C. \( y = \sin(2x + \frac{\pi}{2}) \)

2. Solve a variety of trigonometric equations algebraically over \([0, 2\pi]\).
   A. \( 4\sin^2x = 3 \)  
   B. \( \sqrt{3}\tan x = -1 \)  
   C. \( 2\cos^2x + 5\cosx - 3 = 0 \)

3. Evaluate the limit of a function as \( x \to c \) using any techniques available to you:
   A. \( \lim_{x \to 3} (2x^2 + 11x + 5) \)  
   B. \( \lim_{x \to -5} \frac{2x^3 + 11x + 5}{x + 5} \)
   C. \( \lim_{x \to 0} \frac{\sqrt{x^2 + 3} - 3}{x} \)  
   D. \( \lim_{x \to 0} \frac{1}{\frac{x+2}{2}} \)

4. Find the limit at infinity:
   A. \( \lim_{x \to \infty} \frac{13x^3 + 5x + 1}{x + 2} \)  
   B. \( \lim_{x \to \infty} \frac{15x + 6}{3x^2 - 1} \)  
   C. \( \lim_{x \to \infty} \frac{4x - 3}{\sqrt{5x^2 + 2}} \)
5. Find the equation of the tangent line to:
   A. \( y=3x^2+5 \) at \( x=3 \)
   B. \( y=\tan x \) at \( x=\frac{\pi}{3} \)
   C. \( y=\frac{1}{3}x^2 \) at \( x=64 \)

6. Find the derivative of the following functions using the Product Rule, Quotient Rule, and/or Chain Rule.
   A. \( f(x) = (2x+1)^{\frac{1}{2}}(3x-1)^{-2} \)
   B. \( f(x) = \sqrt{5 - 3x^2} \)
   C. \( f(x) = \frac{5x^2}{\cos x} \)
   D. \( f(x) = \cos^3(2x+1) \)
   E. \( f(x) = \cos(2x+1)^3 \)
   F. \( f(x) = e^{-5x} \cos 3x \)

7. Find the derivative using implicit differentiation.
   A. \( x^2 - y^2 = 16 \)
   B. \( x^2 - xy + y^3 = 8 \)
   C. \( x \sin y - 4y = 11x \)

8. Solve the related rate problems using implicit differentiation.
   A. The side length of a square is changing at 5 cm/sec at the instant when its area is 36 cm². At what rate is the square's area changing at that instant?
   
   B. A 30 foot ladder leaned against a wall begins sliding down such that its base moves away from the wall at 2 feet/min at the instant when its top is 24 feet up the wall. How fast is the ladder moving vertically on the wall?

9. **Set-up** an integral for the volume of the solid formed by revolving bounded by:
   A. \( y=\sqrt{x}, \ y = 0, \ x = 4, \) revolved about the x-axis
   B. \( y=\sqrt{x}, \ y = 0, \ x = 4, \) revolved about line \( x=6 \)
   C. \( y=\sqrt{x}, \ y = x, \) revolved around the y-axis
   D. \( y=\sqrt{x}, \ y = x, \) revolved around the line \( y=-1 \)

10. Evaluate the following indefinite integrals:
    A. \( \int (x^2 + 3x^{-6} - 5\sqrt{x}) \) \( dx \)
    B. \( \int \frac{x^3 - 6x^{-2} + \sqrt{x^3}}{x^\frac{5}{2}} \) \( dx \)
    C. \( \int (3\sin 5x - 5\cos 3x) \) \( dx \)
    D. \( \int x^2(x + 1)^3 \) \( dx \)
11. Evaluate the following definite integrals (sketch a graph of the indicated region)

A. \( \int_{0}^{3} (x^2 - 1) \, dx \)

B. \( \int_{0}^{\sqrt[4]{7}} \frac{2x}{\sqrt{x^4 + 9}} \, dx \)

C. \( \int_{0}^{3} \frac{2x-4}{x^2-4x+5} \, dx \)

**CALCULATOR-ALLOWED PORTION**

12. Estimate the value of \( \int_{0}^{13} f(t) \, dt \) with the table given by using:

<table>
<thead>
<tr>
<th>t</th>
<th>0</th>
<th>4</th>
<th>7</th>
<th>13</th>
</tr>
</thead>
<tbody>
<tr>
<td>f(t)</td>
<td>11</td>
<td>23</td>
<td>14</td>
<td>9</td>
</tr>
</tbody>
</table>

A. LRAM  
B. RRAM  
C. Trapezoids  

13. The derivative of function \( f \) is given by \( f'(x) = 2e^{\sin x} \). For which \( x \) on \((0,9)\) ...

A. does \( f(x) \) have a relative maximum?  
B. does \( f(x) \) have a relative minimum?  
C. does \( f(x) \) have a point of inflection?  
D. is \( f(x) \) decreasing?  
E. is \( f(x) \) concave up?

14. Water enters a reservoir at a rate given by \( R(t) = \arctan(t^2 - 4t + 1) \) gallons/sec for times \( 0 \leq t \leq 9 \). If 18 gallons were in the reservoir at time \( t=0 \), how many gallons will be there at time \( t=7 \)?

15. Brownies are placed in an Easy Bake Oven such that the temperature of the brownies changes at a rate given by \( T'(t) = 1.35 \ln(t^{2.1}+\log(t+2)) \) °F/min for times \( 0 \leq t \leq 15 \). If the brownies have a temperature of 94.5°F at time \( t=14 \), what was their initial temperature at \( t=0 \)?

16. The graph \( f(x) = \frac{4}{x^2-1} \) has

A. one vertical asymptote  
B. the y-axis as the vertical asymptote  
C. the x-axis as the horizontal asymptote and \( x=\pm1 \) as vertical asymptotes  
D. two vertical asymptotes, at \( x=\pm1 \), but no horizontal asymptotes  
E. no asymptotes

17. Find a value for the constants \( k \) and \( m \), if possible, that will make the function, \( f(x) \), continuous everywhere:

\[
\begin{align*}
    f(x) &= \begin{cases} 
        x^2 + 5, & x > 2 \\
        m(x + 1) + k, & -1 < x \leq 2 \\
        2x^3 + x + 7, & x \leq -1 
    \end{cases}
\end{align*}
\]
18. Let \( g(x) \) be the inverse of \( f(x) \). Given the following selected values of \( f(x) \) and \( f'(x) \) what is the value of \( g'(1) \)?

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
<th>( f'(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-2</td>
<td>7</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>4</td>
</tr>
</tbody>
</table>

19. The position of a particle moving along the \( x \)-axis at time \( t \), given by \( x(t) = e^{\cos(2t)}, 0 \leq t \leq \pi \). For which of the following values will \( x'(t) = 0 \)?

A. \( t=0 \)  
B. \( t= \frac{\pi}{2} \)  
C. \( t=0, t=\pi \)  
D. \( t=0, t=\frac{\pi}{2} \)  
E. \( t=0, t=\frac{\pi}{2}, t=\pi \)

20. The first derivative of the function \( f \) is defined by \( f'(x) = \sin(x^3 - x) \) for \( 0 \leq x \leq 2 \). On what intervals is \( f \) increasing?

A. (1,1.445)  
B. (1,1.691)  
C. (1.445,1.875)  
D. (0.577,1.445) \( \cup \) (1.875,2)  
E. (0,1) \( \cup \) (1.691,2)

21. The figure above shows the graph of \( f' \), the derivative of \( f \), on the closed interval \([-1,5]\). The function \( f \) is twice differentiable with \( f'(x) \) having two horizontal tangent lines at \( x=1 \) and \( x=3 \).

A. Find the \( x \)-coordinate of each point of inflection of \( f \) and justify your answer.

B. At what value of \( x \) does \( f \) attain its absolute minimum on the interval \([-1,5]\)? At what value of \( x \) does \( f \) attain its absolute maximum on the interval \([-1,5]\)?

C. Given this is the graph of a particle’s velocity, on what intervals, if any, would it be speeding up and slowing down?